



## Lexicographic Repair Under Querying Prioritised DL-Lite Knowledge Bases

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## الإصلاح الخطي للاستعلام عن قواعد معرفة ذات الأولية بالمنطق الخفيف

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LINK الرابط	RECEIVED الاستقبال	ACCEPTED القبول	PUBLISHED ONLINE النشر الإلكتروني	ASSIGNED TO AN ISSUE الإحالة لعدد
<a href="https://doi.org/10.37575/b/sci/0054">https://doi.org/10.37575/b/sci/0054</a>	16/11/2020	20/01/2021	20/01/2021	01/06/2021
NO. OF WORDS عدد الكلمات	NO. OF PAGES عدد الصفحات	YEAR سنة العدد	VOLUME رقم المجلد	ISSUE رقم العدد
6155	7	2021	22	1

### ABSTRACT

This article discusses the issue of inconsistency in responses from various DL-Lite knowledge bases. This inconsistency problem is at the origin of several sources of assertions with different levels of reliability. The various solutions proposed in the literature that have to do with retrieving an exhaustive and coherent list of responses are not satisfactory from the point of view of reliability and performance. The solution that we present to solve this problem is articulated around two phases: the first phase consists of interrogating the different knowledge bases to retrieve all of the possible answers, which may be inconsistent and/or contradictory, and the second phase consists in repairing these inconsistencies and/or contradictions. To do this, we propose an approach based on three algorithms that we developed in this framework: a first algorithm for non-defeat repair, a second algorithm for lexicographic repair and a third algorithm for non-defeat repair based on lexicography of possible inconsistent responses. The experimental study carried out on the different data collections, as well as the analysis of the results obtained, confirm the performance of our approach as well as its efficiency in regards to productivity and complexity in terms of execution time.

### المخلص

تناقش هذه المقالة مشكلة عدم الاتساق في الاستجابات من مختلف قواعد المعرفة DL-Lite ذات الأولوية. أصل مشكلة التناقض هي المصادر المتعددة للتأكدات بمستويات مختلفة من الموثوقية. الحلول المختلفة المقترحة في الأدبيات من أجل استعادة قائمة شاملة ومتناسكة من الإجابات ليس مرضية من وجهة نظر الموثوقية والأداء. يعتمد الحل الذي نقدمه لحل هذه المشكلة حول مرحلتين: تتكون المرحلة الأولى، أولاً وقبل كل شيء، استجواب قواعد المعرفة المختلفة لاسترداد جميع الإجابات المحتملة التي قد تكون غير متسقة و/ أو متناقضة وتتكون المرحلة الثانية من إصلاح هذا التناقض و/ أو عدم الاتساق. للقيام بذلك، اقترحنا نهجاً يعتمد على ثلاث خوارزميات قمنا بتطويرها في هذا الإطار: خوارزمية أولى للإصلاح غير المهزوم، وخوارزمية ثانية للإصلاح الخطي وخوارزمية ثالثة للإصلاح الخطي غير المهزوم للإجابات غير المتسقة الممكنة. إن الدراسة التجريبية التي أجريت على مجموعات البيانات المختلفة المعتبرة، وكذلك تحليل النتائج التي تم الحصول عليها تؤكد أداء نهجنا وكذلك كفاءته مع الأخذ في الاعتبار الإنتاجية والتعقيد من حيث وقت التنفيذ.

### KEYWORDS

الكلمات المفتاحية

Description logics, conjunctive query, answer profile, inconsistent information

المنطق الوصفي، الاستعلام المترابط، الملف الذاتي للإجابة، معلومات غير متسقة

### CITATION

الإحالة

Abdelmoutia, T., Ghassen, H. and Nazih, O.M. (2021). Al'iislah almuejimi dimn alaistielam ean qawaeid almaerifat DL-Lite dhat al'awlawa 'Lexicographic repair under querying prioritised DL-Lite knowledge bases'. *The Scientific Journal of King Faisal University: Basic and Applied Sciences*, 22(1), 124–30. DOI: 10.37575/b/sci/0054

عبدالمطيع، تلي و غسان، حامدي و نزيه، عمري، محمد. (2021). الإصلاح الخطي للاستعلام عن قواعد معرفة ذات الأولوية بالمنطق الخفيف. *المجلة العلمية لجامعة الملك فيصل: العلوم الأساسية والتطبيقية*, 22(1)، 124-130.

## 1. Introduction

Description logics (DLs) are formal ontological structures for representation and reasoning. Two distinct components are based on a DL knowledge bases (KBs): a terminological basis (called TBox), representing generic knowledge, and an assertional basis (called ABox), containing facts or assertions (Artale, 2009).

Recently, there has been a particular interest in ontology-based data access (OBDA), in which using the TBox provides improved use of the ABox while interrogating it (see Lenzerini, 2011; Poggi et al., 2008).

An ontology is typically checked and validated in such a setting, whereas the assertions may be supplied in large amounts by different and inaccurate sources that may be inconsistent with the ontology, and manually verifying and validating all the assertions is always too costly. This is why reasoning in the face of inconsistency is very important in OBDA.

In an OBDA setting, most works (essentially inspired by approaches in the database field or in sentential logic (Benferhat et al., 1993, 1997; Nebel, 1994) deal with inconsistency in KBs by proposing multiple inferences, called semantics. These semantics depend on the idea of a maximally assertional fix that firmly identifies with the thought of a fix from an information base (Lembo et al., 2010) or a

maximally consistent subset using propositional logic (see Brewka, 1989; Rescher and Monor, 1970). A repair of an ABox is merely an assertional subset consistent with ontology. Assertions are also provided by multiple and possibly contradictory sources with different standards of reliability in many applications. A certain origin may have various collections of conflicting information with different levels of trust. These information sets provide an inconsistent and prioritised knowledge base (ABox).

The following statements summarise the contributions of this paper:

1. The first contribution of this paper is to present a description of the inconsistency issue in the DL-Lite knowledge bases,
2. The second contribution tackled in this work is a review of the literature on inconsistent and prioritised DL-Lite knowledge base problems,
3. Finally, a novel approach is empirically developed to repair answers under inconsistent and prioritised DL-Lite knowledge bases, which has not been addressed in previous scientific literature.

We organise the rest of this article as follows: we give the related works in Section 2. Then, we give the needed background about prioritised DL-Lite KBs in Section 3. In Section 4, we present the basic concepts of consistent unified answer repair. In Sections 5, we show three approaches to handling an inconsistent knowledge base. We provide our experimental analysis in Section 6, and we conclude the paper in Section 7.

## 2. Related Works

There are many scientific works that have dealt with the problem of inconsistency in DL-Lite KBs, which we can summarise in seven groups, as follows:

1. The works that deal with propositional setting, (Benferhat et al., 1995) where the authors studied the function of priorities in the management of inconsistency within the framework of sentential logic by incorporating acceptable relationships with implications capable of inferring non-trivial assumptions;
2. The works that deal with different semantics, presented essentially in the works of Trivela et al. (2019), Bienvenu et al. (2019) and Dixit (2019). Trivela et al. (2019) suggest a general structure that takes into account the semantics of Intersection Closed ABox Repair (ICAR). Bienvenu et al. (2019) contribute a practical approach for computing the query answers under three semantics, ABox Repair (AR), Intersection ABox Repair (IAR) and brave semantics in the lightweight description logic *DL-Lite<sub>R</sub>*. However, Dixit (2019) suggests Consistent Answering via Satisfiability (CAvSAT) as a new solution to inconsistency response problems;
3. The works that deal with standard *DL-Lite* knowledge bases (Bertossi, 2011; Bienvenu and Rossati, 2013). The authors adapted many inconsistency algorithms via some provided standards of consistency in many applications, and they studied the data complexity of conjunctive query answering under the standard *DL-Lite* knowledge bases;
4. The works that deal with prioritised *DL-Lite* knowledge bases; particularly, the works of Lembo et al. (2010, 2015), Bienvenu and Rossati (2013), Lenzerini (2011) and Benferhat et al. (2016), which compute the consistent subsets of assertions (repairs) in order to restore the consistency of *DL-Lite* knowledge bases. In addition, Benferhat et al. (2015, 2016) propose a new non-objection inference relation based on the option of only one preferred repair and discuss the complexity of computation inference on ideal repair. Additionally, Telli et al. (2017) propose polynomial strategies for finding a unification repair consistent under all the *DL-Lite* knowledge bases;
5. The work that deals with OBDA settings, such as Bienvenu et al. (2014)'s, which is one of few works in this context and which focuses on inference with prioritised *DL-Lite* knowledge bases in OBDA settings;
6. The works that do not deal directly with KBs, such as the works of Hamdi et al. (2018), Boughammoura et al. (2012, 2015) and Boughammoura and Omri (2017), in which the authors studied several query answering strategies and proposed new approaches, querying responses from hidden datasets in which *DL-Lite* describes KB;
7. The works that deal with answers querying, such as the works of Artale et al. (2009) and Staworko et al. (2012), in which the authors explore the principle of priority answering inconsistent *DL-Lite* knowledge bases by using user preferences to limit the set of repairs down to a set of preferred repairs.

Our work belongs in a group dealing with answers querying under prioritised DL-Lite knowledge bases. For this purpose, an algorithm starts by querying each stratum of ABox in order to provide us with all the possible response sets. After that, once the response sets are consistent, no repair is applied. Otherwise, the algorithm repairs the response sets.

In addition, this paper uses DL-Lite Du and Shen (2013) due to its efficiency in conjunctive query answering and computing contradictory knowledge. Also, we relied only on conjunctive query, as it includes all basic queries, and many first-order queries can be written as conjunctive queries.

## 3. DL-Lite Knowledge Base

The DL-Lite family (Artale et al., 2009; Poggi et al., 2008) is included in OWL2 QL syntax. The knowledge representation format for DL-Lite is as follows: NC is a set of atomic concepts, NR is a set of atomic roles and NI is a set of individuals or assertions. We consider three

connectors ' $\neg$ ', ' $\exists$ ' and ' $\bar{\cdot}$ ', which are used to describe complex concepts and complex roles as follows (Artale et al., 2009):

$$R_1 \rightarrow R_2 \text{ or } R_1 \rightarrow R_1^-$$

$$E \rightarrow R_1 \text{ or } E \rightarrow \neg R_1$$

$$Bc \rightarrow Dc \text{ or } B \rightarrow \exists R_1$$

$$Cc \rightarrow Bc \text{ or } Cc \rightarrow \neg Bc$$

such that Dc represents an atomic concept, R1 represents an atomic role and R1- represents the inverse of R1. However, Bc represents basic concept, Cc represents complex concept, R1 represents basic role and E represents complex role. A DL-Litecore knowledge base K is a pair  $K = \langle T, A \rangle$  (Benferhat et al., 1997).  $T = TBox$  is made up of a finite set of inclusion axioms between concepts of the form  $B \subseteq C$  or  $B \subseteq \neg C$ .  $A = ABox$  contains the finite set of assertions (facts) of atomic concepts and roles of the form  $D(a)$  and  $P(a, b)$ .

The DL-LiteF language extends DL-Litecore with the capability of functional specification on roles or their inverses of the form (functR). The DL-LiteR language extends DL-Litecore with the ability to specify inclusion axioms between roles in TBox of the form  $R \subseteq E$ . Note that DL-Lite language does not use connective or disjunctive operators. However, a logical transformation makes it possible to obtain conjunctions and disjunctions as follows:

- A conjunction of the form  $B \subseteq C \cap D$  is equivalent to the pair of inclusion axioms  $B \subseteq C$  and  $B \subseteq D$ ;
- A disjunction of the form  $C \cup D \subseteq B$  is equivalent to the pair of inclusion axioms  $C \subseteq B$  and  $D \subseteq B$ .

Note that all DL-Lite knowledge bases can be written as a First-Order Logic (FOL) knowledge base.

In addition, we share the semantics of DL-Lite knowledge bases. A semantics is an interpretation  $I = (\Delta^I, \cdot^I)$  from a non-empty domain  $\Delta^I$  to interpret function  $\cdot^I$  such that:

$$\{\forall x \in \mathbb{N}I, \exists xI \in \Delta^I, \forall C \in \mathbb{N}C, \exists CI \subseteq \Delta^I \text{ and } \forall R \in \mathbb{N}R \text{ and } \exists R \subseteq \Delta^I \times \Delta^I\}.$$

Note that a DL-Lite knowledge base is inconsistent if it does not admit any model.

### 3.1. Prioritised Profile DL-Lite Knowledge Base:

We claim that the prioritised profile DL-Lite knowledge base  $K_P = \langle T, P_S \rangle$  for all sets of prioritised ABoxes such that T is a flat (standard) DL-Lite TBox, and  $P_S = \{L_1, \dots, L_m\}$  is a prioritised ABox profile, where  $L_i$  is a layer (stratum)  $i$ , which includes a list of assertions that have the same level of priority, and  $\forall j > i$   $L_i$  is more important than  $L_j$ .

### 3.2. Conjunctive Query:

We claim that the query  $Q = \{f(v)\}$  is a First-Order Logic formula when:

- $(v) = (v_1, \dots, v_n)$ : free variables,
- $n$ : the arity of  $Q$  and atoms of  $f(v)$ ,
- $f(v): D(t_i) \text{ or } R(t_i; t_j), \forall D \in \mathbb{N}C \text{ and } \forall R \in \mathbb{N}R \text{ and } t_i, t_j \text{ are terms.}$

Note that  $f(v)$ , of the form  $\exists (w). \text{conj}(w; w)$ , and  $w$  are existentially quantified variables, and  $\text{conj}(x; y)$  is a conjunction of atoms of the form  $D(t_i)$  or  $P(t_i; t_j)$ .  $Q$  is said to be a conjunctive query (CQ). An answer to a CQ  $Q(x) \leftarrow \text{conj}(x; y)$  over  $K = \langle T, A \rangle$  is a non-empty set of tuples  $s = (s_1, \dots, s_k) \in \mathbb{N}I \times \dots \times \mathbb{N}I$  such that  $\langle T, A \rangle \models Q(s)$ .

Now, let  $Q(x)$  be a conjunctive query. We consider  $SP_s = \{S_1, \dots, S_m\}$  a set of responses about  $Q(x)$  under  $P_s$ , and  $S_i = s \in \mathbb{N}I \times \dots \times \mathbb{N}I : \langle T, A \rangle \models Q(s)$  with certainty, when there is no answer to the query  $Q(x)$  with respect to  $L_i, S_i = \emptyset$ .

**Example 1:** We consider  $K = \langle T, P_S \rangle$  to be a prioritised DL-Lite

knowledge base.

Thus, we have

$$T = \{C1 \subseteq \neg C2, C2 \subseteq \neg C3, R1 \subseteq R2 \text{ and } P_s = \{L1, L2, L3\},$$

where

$$L1 = \{C2(a), R1(a, c), C1(a)\},$$

$$L2 = \{C1(b), R1(b, c), C3(e), R2(e, c)\} \text{ and}$$

$$L3 = \{C3(b), R1(b, c), C3(a), R1(a, c)\}.$$

We consider also the following conjunctive query  $Q_c$ , which requires any individual  $v$  have a relationship to  $c$  through the role  $R1$ :  $Q(v) =$

$\exists v: R1(v, c)$ . The list of responses  $QPs$  to this query are

$$QL1 = \{C1(a), C2(a)\},$$

$$QL2 = \{C1(b)\} \text{ and}$$

$$QL3 = \{C3(b), C3(a)\}.$$

#### 4. Consistent Unified Answers Reasoning

In this section, we give a brief refresher, proposed by Benferhat et al. (1992). Then, we will use these concepts to present our approach.

The Conflict Answer Sets ( $CA \subseteq (QPs)$ ) represent a minimal inconsistent subset  $CA$  of the assertions associated with  $SPs$  such that  $CA$  is inconsistent. Hence,  $\forall \alpha \in CA, CA \setminus \{\alpha\}$  is consistent with respect to  $TBox$ .

The Free Answer Sets (free(QPs)) represent the subset of facts  $\in QPs$  that are not included with the Conflict Answer Sets  $CA$  in  $QPs$  with respect to  $TBox$ . We say that an assertion  $\beta \in QPs$  is free if  $\forall \alpha \in C(QPs): \beta \notin \alpha$ . The free individual in a propositional logic context has been previously suggested by Lutz (2013).

The answers repair  $RA \subseteq (QL1, \dots, QLm)$  is a consistent subset, and it is denoted by MARA (maximally inclusion-based answers repair of QPs) if:

$\langle T, RA \rangle$  is consistent and

$$\forall RA \subseteq (QL1, \dots, QLm): RA \notin R'A, R'A \text{ is inconsistent.}$$

This definition of MARA is similar to that defined in Lembo et al. (2010).

We denote MARA(QPs) by the set of MARA of QPs with respect to  $T$ . Inconsistency in flat DL-Lite KBs can be accomplished using the principle of answers repair by applying standard request answers using the entire set of answers repair (AR-entailment (Lembo et al., 2010) or using only one answers repair.

The preferred inclusion-based answers repair RARA(QPs) is the extension of the MARA definition, when the DL-Lite ABox is prioritised (Bienvenu et al., 2014).

$$PARA(QPs) = PA1 \cup \dots \cup PAm \text{ of } QPs$$

such that:

$$\nexists MARA(QPs): P'A1 \cup \dots \cup P'Am \text{ of } QPs, \text{ and}$$

$$\text{if } i \text{ is an integer: } PAi \in P'Ai, \forall j=1, \dots, (i-1), PAj = P'Aj.$$

**Example 2:** We continue from the previous example.

According to the definition of the Conflict Answer Sets, we have

$$CA(QPs) = \{C1(a); C2(a)\}, \{C2(a); C3(a)\}.$$

According to the definition of the Free Set Answers, we have

$$\text{free}(QPs) = \{C1(b); C3(b)\}.$$

While, according to the definition of the preferred inclusion-based answers repair, we have

$$PARA(QPs) = \{C1(a), C1(b), C3(b)\}.$$

The next section discusses a repair that is used as a selection of facts to deal with inconsistent answers and propose new approaches

based on a lexicographic approach. The result of the given approaches is a consistent set of corresponding answers.

## 5. Approaches for Repairing Answers Profile

For the rest of the paper, we consider the following settings:

- $K_P = \langle T, P_s \rangle$  as a prioritised DL-Lite knowledge base;
- $P_s = \{L_1, \dots, L_m\}$  as a prioritised ABox profile;
- $Q$  as a conjunctive query;
- $S_{P_s} = \{S_1, \dots, S_m\}$  as a set of answers to a query  $Q$  with respect to  $P_s$ ;
- $Q_{P_s} = \{Q_{L_1}, \dots, Q_{L_m}\}$  as a set of assertions associated with  $S_{P_s}$ ;
- $Q_{L_i} = \{Q(s): s \in S_{ij}\}$ , where  $Q_{L_i}$  is a set of answers to the query  $Q$  for each  $L_i$ .

For a given repair of answers, we start by interrogating each ABox with a conjunctive query. Then, we recover answers for each level of ABox. Finally, one of the proposed approaches will be run to repair the total set of responses. Algorithm 1 presents the steps necessary to obtain consistent unified answers under a prioritised profile DL-Lite knowledge base.

**Algorithm 1: Response processing from inconsistent DL-Lite knowledge bases**

```

Data: A prioritised DL-Lite KBs  $\langle T, \{L_1, \dots, L_m\} \rangle$ 
Conjunctive query  $Q$ 
Result: Consistent unified answers
Interrogation of each ABox  $L_i$  by  $Q$ 
Recovery answers for each ABox  $\langle T, \{Q_{L_1}, \dots, Q_{L_m}\} \rangle$ 
If  $\langle T, \{Q_{L_1}, \dots, Q_{L_m}\} \rangle$  is consistent, then
Consistent unified answers are returned
Or
Repairing using one of the proposed approaches
Returns consistent unified answers
End.
```

### 5.1. Non-Defeated Repair of Inconsistent Answers

This new repair of inconsistent answers consists of evaluating the set of assertions associated with the answers to a given query in reference to the ABox profile proposed by Benferhat et al. (1992). The non-defeated repair of inconsistent responses is  $ndA(QPs) = A'1 \cup \dots \cup A'm$ , where

$$\forall i = 1 \dots m; A'i = \text{free}(QL1 \cup \dots \cup QL_i);$$

namely,  $ndA(QPs) = \text{free}(QL1) \cup \text{free}(QL1 \cup QL2) \cup \dots \cup \text{free}(QL1 \cup \dots \cup QLm)$ .

The non-defeated answers repair is computed in polynomial time in DL-Lite, as it appears in Algorithm 2. It starts by initialising the set of  $ndA$  repair. Then, it computes the set of free assertions in  $(QL1 \cup \dots \cup QL_i)$ , which is done in polynomial time because the free set assertions can be computed in linear time with respect to the conflict set assertions. Hence, the computation of  $ndA(QPs)$  is also done in polynomial time.

**Algorithm 2: Non-Defeated Repair of Inconsistent Answers**

```

Data: A prioritised DL-Lite KBs  $\langle T, \{L_1, \dots, L_m\} \rangle$ 
Conjunctive query  $Q$ 
Result: Consistent unified answers  $ndA(QPs)$ 
Interrogation of each ABox  $L_i$  by  $Q$ 
Recovery answers for each ABox  $\langle T, \{QL_1, \dots, QL_m\} \rangle$ 
 $nd_A(Q_{P_s}) \leftarrow \phi$ 
for  $i=1$  to  $m$  do
 $nd_A(Q_{P_s}) \leftarrow nd_A(Q_{P_s}) \cup \text{free}(Q_{L_i} \cup \dots \cup Q_{L_i})$ 
return  $ndA(QPs)$ 
end.
```

**Example 3:** We continue from the previous example.

We have  $\text{free}(QL1) = \{\emptyset\}$ ,  $\text{free}(QL2) = \{C1(b)\}$  and  $\text{free}(QL3) = \{C3(b), C3(a)\}$ .

Hence,  $ndA(QPs) = \{C1(b); C3(b); C3(a)\}$ .

Clearly, A query  $Q$  is said to be a  $ndA(QPs)$ -the consequence of  $KPs$ , denoted by  $KPs \models Q$ , if and only if  $KP = \langle T, ndA(QPs) \rangle \models Q$ .

### 5.2. Lexicographic Repair of Inconsistent Answers

In the propositional context, lexicographic inference has been commonly used by Benferhat et al. (1993). One approach to

lexicographic repair is PARLex(QPs) of inconsistent answers, which is based on the cardinality criterion instead of the set inclusion criterion. The lexicographic repair of inconsistent answers PARLex(QPs) is defined as follows:

$\forall PQPs = PA1 \cup \dots \cup PAm \in \text{PARA}(QPs): \nexists i$  such that  $|PAi| > |LAI|$  and  $\forall j < i, |PAj| = |LAI|$ , where  $|X|$  represent the cardinality of the set X.

Clearly, using a lexicographic-based approach comes down to selecting among the set of repairs in PARA(QPs), particularly the ones having the maximal number of elements (See Algorithm 3).

**Algorithm 3: Lexicographic Repair of Inconsistent Answers**  
 Data: A prioritised DL-Lite KBs  $\langle T, (L1, \dots, Lm) \rangle$   
 Conjunctive query Q  
 Result: Consistent unified answers PARLex(QPs)  
 Interrogation of each ABox Li by Q  
 Recovery answers for each ABox  $\langle T, (QL1, \dots, QLm) \rangle$   
 $PAR_{Lex}(Q_{Ps}) \leftarrow \emptyset$   
 for  $i=1$  to  $m$  do  
 $PAR_{Lex}(Q_{Ps}) \leftarrow PAR_{Lex}(Q_{Ps}) \cup PAR(Q_{Li} \cup \dots \cup Q_{Li})$   
 return PARLex(QPs)  
 end.

In particular, the large number of PARLex(QPs) that can be determined from an inconsistent DL-Lite knowledge base is one of the main PAR-entailment issues. However, Algorithm 3 proceeds from the first answers set to the less preferred ones. Thus, the assertions selected do not conflict with the ones in the first answers set in order to ensure the maximally prioritised returned set with respect to lexicographic ordering. Hence, this algorithm is based on checking for inconsistency, and its computational complexity is polynomial.

**Example 4:** We continue from the previous example.

We have PARLex(QPs) = {C2(a), C1(b), C3(b)}. Clearly, a query Q is said to be a PARLex(QPs)-the consequence of KPs, denoted by KPs  $\models Q$ , if and only if  $KP = \langle T, \text{PARLex}(QPs) \rangle \models Q$ .

The next subsection proposes an approach that consists of introducing a cardinality criterion instead of a set inclusion criterion and is based on non-defeated repair of inconsistent answers. We discuss lexicographic-based non-defeated repair of inconsistent answers.

### 5.3. Lexicographic-Based Non-Defeated Repair of Inconsistent Answers

The lexicographic-based non-defeated repair of inconsistent answers denoted by

$ndLex(QPs) = L'1 \cup \dots \cup L'm$  is defined as follows:

$\forall i = 1$  to  $m: Li = \bigcap RA \in \text{MARALex}(L1 \cup \dots \cup Li) RA$ ,

where  $\text{MARALex}(Li) = RA: RA \in \text{MARA}(Li)$ , and  $\nexists R'A \in \text{MARA}(L)$  such that  $|R'A| > |RA|$ .

**Algorithm 4: Lexicographic-Based Non-Defeated Repair of Inconsistent Answers**  
 Data: A prioritised DL-Lite KBs  $\langle T, (L1, \dots, Lm) \rangle$   
 Conjunctive query Q  
 Result: Consistent unified answers PARLex(QPs)  
 Interrogation of each ABox Li by Q  
 Recovery answers for each ABox  $\langle T, (QL1, \dots, QLm) \rangle$   
 $nd_{Lex}(Q_{Ps}) \leftarrow \emptyset$   
 for  $i=1$  to  $m$  do  
 $nd_{Lex}(Q_{Ps}) \leftarrow nd_{Lex}(Q_{Ps}) \cup PAR_{Lex}(Q_{Li})$   
 return PARLex(QPs)  
 end.

According to the Algorithm 4, the main advantage of the lexicographic-based non-defeated repair of inconsistent answers approach is the production of more conclusions than the standard non-defeated repair of inconsistent answers approach, ndA(QPs). Note that this algorithm is based on the previous polynomial algorithms. Hence, it is also done in polynomial time.

**Example 5:** We continue from the previous example.

We have  $ndLex(QPs) = \{C1(a); C1(b); C3(b); C3(a)\}$ .

Clearly, a query Q is said to be a ndLex(QPs)-the consequence of KPs denoted by KPs  $\models Q$ , if and only if  $KP = \langle T, ndLex(QPs) \rangle \models Q$ .

## 6. Experimental Analysis

This section presents an experimental analysis of the running time and productivity of our proposed approaches.

### 6.1. Software and Hardware Environments

We have implemented our algorithms to compute a consistent unified answer in Java programming language, Web Ontology Language Second Edition-Query Language (OWL2-QL) function syntax and a Structured Query Language Lite (SQLite) database engine for relational database manipulation. Then, we used:

- The benchmark existing on <https://code.google.com/p/combo-obda>;
- The TBox of ontology Lehigh University Benchmark  $\exists$  (LUBM $\exists$ ) 20 (Calvanese et al., 2005) and
- The Extended University Data Generator (EUDG) for generating the ABoxes.

All experiments were performed on an ASUS Sonic Master Introduction laptop Model X556QUK with an Intel (R) Core(TM) (i5) 7200 CPU @ 2.50 GHz 2.71 GHz. 4 GB DDR3 RAM. This hardware configuration is installed in a 64-bit operating system, x64 processor (Windows 10 Home).

### 6.2. Experimental Parameters

The theoretical foundations of this work can be found in Calvanese et al. (2005)'s document. Their work explains how exactly we modified the original LUBM data generator and ontology. It consists an evaluation of the ABox, which stores relational database (DB) queries expressed from the negative closure of the TBox to exhibit whether the KBs contains conflicting elements. This negative closure of a KBs is made of the list of all negative axioms of the form  $(B \subseteq \neg C)$ , which can be derived from TBox by applying positive rules onto negative ones. The LUBM $\exists$  20 ontology contains the axioms presented in Table I. Our proposed approaches are based on a DL-Lite ontology parser and an SQLite database engine. They are also specifically focused on some of the following operations: checking consistency and checking conflict.

We emphasised that the process of computing the conflicts set would be realised once and for all and kept it in mind during all experimentations. After all the settings were available, we proceeded to repair the DL-Lite knowledge base using the different approaches proposed in this work.

Table 1. Ontology LUBM Information

Axioms	Size	Examples
Classes	129	FullProfessor, Faculty
Object Property	28	HasFaculty, isPartOfUniversity
Data Property	7	DataPropertyDomain(age Person)
Subclass Of	153	SubclassOf(Professor Faculty)
Disjoint Class Of	643	DisjointClassOf (Teacher, Student)
Subobject Property Of	5	SubobjectPropertyOf(headOf worksFor)
Inverse Object Properties	3	InverseObjectProperties(HasSupervisor)
Disjoint Object Properties	227	DisjointObjectPropertyOf(headOf advisor)
Object Property Domain	25	ObjectPropertyDomain(advisor Person)
Object Property Range	22	ObjectPropertyRange(advisor Professor)
Data Property Domain	4	DataPropertyDomain(age Person)

### 6.3. Tests and Results

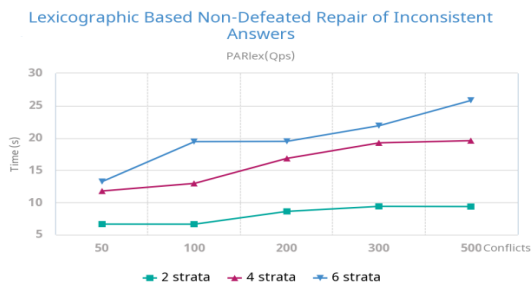
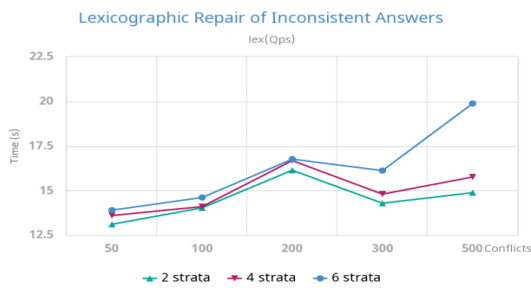
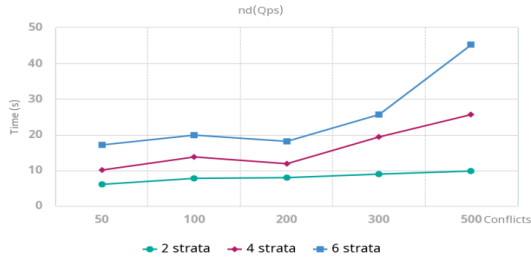
We used an SQLite engine to calculate the conflict elements and to check for inconsistencies. This allowed for efficient management of inconsistency. Then, using EUDG, we generated an ABox, divided it into 2, 4 and 6 strata, each with 50, 100, 200, 300 and 500 sets of conflict elements. For each case, we launched conjunctive query, and we collected the corresponding answers. We focused on two important features: calculation time and productivity of answers repair to evaluate our proposed algorithms. We analysed our

approaches by evaluating the results observed in the following graphs:

### 6.3.1. Calculation Time of Answers Repair

By calculation time of answers repair, we mean the time it took to compute our proposed algorithms of repairs. The set of graphs in Fig. 1 presents the results obtained in these experimentations.

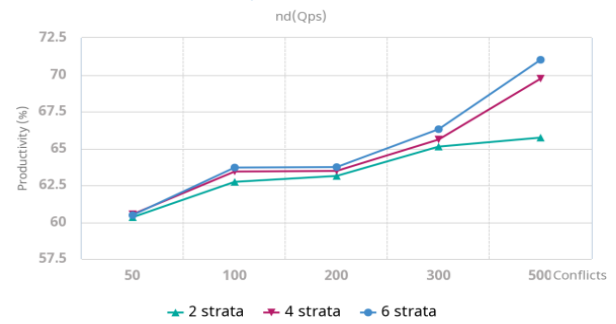
Fig. 1. The time (in seconds) taken to calculate the answers repair Non-defeated Repair of Inconsistent Answers



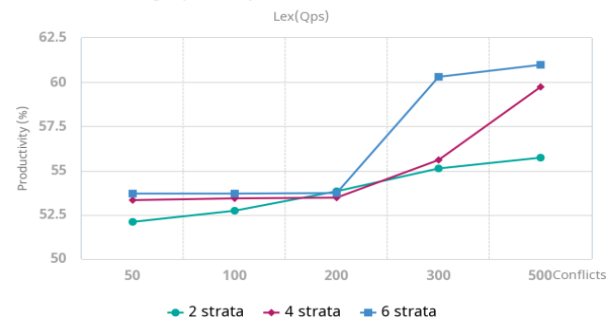
### 6.3.2 Productivity of Answers Repair

By productivity, we mean all assertions that were retained from the answers that restore the answers' consistency. The results obtained in this experiment are presented in Fig. 2.

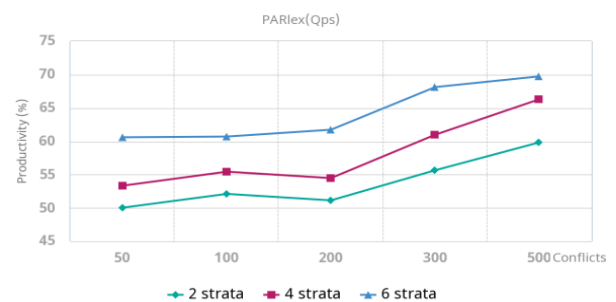
Fig. 2. The assertions retained after the answers repair (the productivity in %) Non-Defeated Repair of Inconsistent Answers



Lexicographic Repair of Inconsistent Answers



Lexicographic Based Non-Defeated Repair of Inconsistent Answers



### 6.4. Analysis and Evaluation of Results

According to Fig 1, the running time to calculate the lexicographic-based answers repair using the non-defeated approach was greater than the lexicographic and non-defeated approaches. Thus, the running time was affected by increasing the number of conflicts and the number of strata.

Similarly, the productivity of the answers repair shown in Fig 2 proves that the lexicographic-based non-defeated answers repair algorithm was more productive than the lexicographic and non-defeated approaches in all cases. Consequently, the productivity was affected proportionately by the size of the conflicts set and the number of strata in the ABox.

More precisely, there was a convergence between all proposed approaches to running time and productivity when the size of the conflicts set was less than 200 elements. Nevertheless, we found significant increases in running time and productivity after that.

These results proved that the productivity (number of answers returned by applying query) of an applied lexicographic base to non-defeated answers repair on prioritised KBs was more than the repairs standard approach to non-defeated answers repair.

Generally, there was a high degree of efficiency when the

correspondence between the query and the number of requested assertions was significant. We also noticed that the lexicographic-based non-defeated approach was the best performing repair approach, compared to the other two approaches.

Finally, we noted that the calculations of conflict sets had an important impact on our approaches due to having to continuously update the degree of inconsistency, which can be increased when new assertions are added. Hence, it suffices to account for the new conflicts that arise as a result of adding new assertions.

Our investigation yielded some positive results, including the development of new polynomial algorithms, such as lexicographic-based non-defeated answers repair with conjunctive query. The analysis we conducted for consistent query answering in lightweight ontologies proved that our repairs were computed incrementally, starting from the first layer until the last one.

In summary, the size of conflicting elements and the number of layers in the ABoxes are considered principal properties that directly influence the running time and the productivity of our approaches. Particularly, the lexicographic-based ndA-inference offers an important number of consistent answers compared to the ndA-inference and the Lex-inference.

## 7. Conclusion

In this article, we focused on the problem of inconsistent answers from prioritised DL-Lite KB. After giving the basic concepts of DL-Lite hierarchical knowledge bases and how to fix inconsistent responses of conjunctive queries, we presented an approach which relies on three algorithms in order to repair all possible answers, instead of fixing all the knowledge bases.

Our approach consisted of, first, a non-defeated repair, then a lexicographic repair and, finally, a non-defeated repair based on the lexicographic of possible incoherent responses. These different repairs are done in a polynomial time, which allowed us to reduce the operating complexity and increase productivity while remaining efficient, as shown in the paragraph which presents the experimental study.

In the future, we will expand interest through the following axes. The first axis is to compare in-depth our approach, and the main approaches in the literature, with other data collections in order to give researchers more studies and analyses about handling the problem of repairing inconsistent and/or contradictory responses in the context of priority queries in DL-Lite knowledge bases. Through this study, we hope to definitively confirm the performance and robustness of our proposed approach. As a second axis, we plan to study how to deal with missing information (conflict answer sets) in knowledge bases. And as a third axis, we will add how to repair knowledge bases starting with repairing answers.

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